# Running time supplements: energy-efficient train control versus robust timetables

Gerben M. Scheepmaker [a,b,1], Rob M.P. Goverde [a]
[a] Department of Transport and Planning, Delft University of Technology
P.O. Box 5048, 2600 GA Delft, The Netherlands
[b] Department of Logistics, Netherlands Railways (NS)
P.O. Box 2025, 3500 HA Utrecht, The Netherlands
[1] E-mail: g.m.scheepmaker@tudelft.nl, phone: +31 (0) 15 27 84914

**Abstract**

Energy-efficient train operation is not yet included in the timetable design process in the Netherlands. Hence, running time supplements are not optimally distributed in the timetable. Therefore research has been conducted on the possibilities to better incorporate energy-efficient train operation into the railway timetable. This paper describes the developed *EZR model* (energy-efficient operation or in Dutch 'EnergieZuinig Rijden') based on optimal control theory and algorithm that determines the joint optimal cruising speed and coasting point for individual train trips; taking into account a desired robustness, the possibilities for energy-efficient operation, and the desired punctuality during operations. The model is applied on a case study on a sprinter/local train line in the Netherlands between Utrecht Centraal and Rhenen. The model results show that it is better to distribute the running time supplements evenly than concentrating it near the main stations.

**Keywords**
Optimal train control, energy minimization, timetabling

## 1  Introduction

Energy consumption is an important topic nowadays, also in the railway sector. A lot of money can be saved by decreasing the energy consumption of the trains. Besides there are external benefits like decrease in $CO_2$ emission and noise hindrance. At the Netherlands Railways (NS), the largest passenger Railway Undertaking (RU) in the Netherlands, a lot of attention is paid to energy-efficient train driving. At this moment, NS is using the so-called *UZI method* (Universal energy-efficient driving idea or in Dutch 'Universeel Zuinig rijden Idee') during training of all train drivers (Franke, 2012). The *UZI method* is developed by train driver Freddy Velthuizen of NS. This method advice the train driver to accelerate as fast as possible until the recommended speed is reached. Then the train starts to coast at the right point so the train will reach the next station in time. Hence, the *UZI method* describes where the train driver should start coasting.

Table 1: *UZI method* for a short or long distance (Franke, 2012)

| Short distance | | Long distance | |
|---|---|---|---|
| **Running time [min]** | **Coasting speed [km/h]** | **Speed limit [km/h]** | **Coasting time before arrival [min]** |
| 2 | 80 | 140 | 8 (not for SGM) |
| 3 | 90 | 130 | 7 (not for SGM) |
| 4 | 100 | 120 | 6 |
| 5 | 110 | 110 | 5 |
| 6 | 120 | 100 | 4 |
| 7 | 130 (not for SGM) | | |
| 8 | 140 (not for SGM) | | |

The basic *UZI method* that is currently taught to train drivers distinguishes between a short distance (running time between two stopping stations is maximum 8 minutes) where the running time determines the coasting point and a long distance (running time between two stopping stations is more than 8 minutes) where the speed limit determines the time where to start coasting. The method is visualized in Table 1. The method can be applied at every type of rolling stock of NS (universal). This leads to considerable energy-savings of about 2% to 5% per year.

However the current method applied at NS is not the most energy-efficient train operation. Nowadays a lot of research is done on the topic of Driver Advisory Systems (DAS), that gives the train driver speed advice in order to minimize energy consumption and increase punctuality. A good overview of different DAS systems can be found in Kent (2009) and Panou, Tzieropoulos, and Emery (2013). The energy-efficient driving strategy is determined by optimal control theory which leads to the theoretical optimum. In literature a lot of research has been done since 1968. Especially a lot of research on both continuous and discrete optimal control has been done by the Signalling and Control Group of the University of South Australia during the last thirty years, which can be found in for example Howlett and Pudney (1995), Howlett (2000) and Howlett, Pudney, and Vu (2009). Their developed algorithms have been implemented in different DAS like *Metromiser, Freightmiser* and *Energymiser* (A. R. Albrecht, Howlett, Pudney, & Vu, 2013; Howlett, 1990; Howlett et al., 2009).

All those strategies are in principle based on Pontryagin's Maximum Principle which derives the optimal train control. There are three different types of algorithms to solve the problem of energy-efficient train control between two stops:
1.  Differential equations: modelling the train driving between two stops as a dynamic state model and explicitly find the solution of the optimal control by solving the differential equations. This will lead to the exact solution of the control problem. However solving the set of different differential equations analytically is a very difficult process, so different algorithms are used to solve the control problem.
2.  Artificial intelligence or searching algorithms: the driving behaviour of trains is modelled as a dynamic model and the (sub)optimal solution is estimated with artificial intelligence or searching algorithms using knowledge of the optimal control regimes. Examples can be found in Chevrier, Pellegrini, and Rodriguez (2013) and Sicre, Cucala, and Fernández-Cardador (2014).
3.  Simulation: the behaviour of trains is modelled as a non-linear model and the optimal control is estimated using simulated of the knowledge of the optimal control regimes. Domínguez, Fernández-Cardador, Cucala, and Pecharromán (2012) give an example of simulation.

It is also possible to solve the problem over multiple stations. This can be done by for example Dynamic Programming (DP) of Bellman. In this method the train driving is simulated between two stations, by assuming a multistage decision process and it finds the optimal solution with DP. The method uses the differential equations to solve the energy-efficient train control. An example can be found in T. Albrecht and Oettich (2002).

Energy-efficient train operation is related to the timetable, because the running time supplements in the timetable determine how much time there is for energy-efficient train operation. Running time supplements are the running times above the technical minimum running time between two timetable points. These supplements are primary aimed to cope with variations in running times and can also be used to recover from small delays. If a train drives punctual then these supplements can be used for energy-efficient train operation. However, energy-efficient train operation is mostly not considered during timetable design. Recently in the Netherlands, the running time supplements were allocated mostly near main stations where punctuality is measured; the so called tightening of the timetable (in Dutch 'straktrekken'). This means that running times on part of the line are experienced as very tight by the train drivers. Moreover, times at timetable points are rounded to the closest whole minutes (<0.5 min rounded down and ≥ 0.5 min rounded up). This rounding can lead to unrealisable times at the timetable points. The combined effect of tightening and rounding of the timetable leads to unrealistic running times for the train driver. This can hinder surrounding trains and energy-efficient train operation is not possible.

Energy-efficient train operation is thus not yet included in the timetable design process in the Netherlands. Hence, running time supplements are not optimally distributed in the timetable. Commissioned by the Netherlands Railways (NS) and in collaboration with Delft University of Technology (TU Delft) research has been conducted on the possibilities to better incorporate energy-efficient train operation into the railway timetable. This paper describes the developed *EZR model* (energy-efficient operation or in Dutch 'EnergieZuinig Rijden') and the algorithms that determines the joint optimal cruising speed and coasting point for individual train trips; taking into account a desired robustness of the timetable, the possibilities for energy-efficient operation, and the desired punctuality during operation. Because it was important to find the energy optimal solution, the applied *EZR model* uses both differential equations and searching algorithms to find the optimum, like the papers of Howlett et al. (2009), Khmelnitsky (2000) and Liu and Golovitcher (2003). The model is also applied to compare the *UZI method* of NS with the theoretical optimum. Extended research results can be found in Scheepmaker (2013).

The paper is structured as follows. Chapter 2 gives a model description of the energy-efficient train control based on Pontryagin's Maximum Principle and the differential equations. Then this basic description is used to describe the algorithm applied in this research. This is done in chapter 3. The model has been applied on a case study to test its effectiveness, which is discussed in chapter 4. Based on the model results, conclusions are drawn in chapter 5.

## 2   Model description

This section describes the model for energy-efficient train control. Several authors have described the basic energy-efficient driving model, like T. Albrecht (2008), Howlett et al. (2009) and Liu and Golovitcher (2003). The description in this paper is based on the model described in Scheepmaker (2013).

Throughout the paper we consider the following assumptions:
- The train is modelled as point mass (Brünger & Dahlhaus, 2008).
- The *EZR model* takes into account gradients, curves and tunnels.
- The *EZR model* optimizes per section with constant maximum speed with only one coasting moment.
- Regenerative braking is not taken into account, since the voltage on the catenary in the Netherlands is relatively low compared to other countries (1.5 kV DC). Hence, we assume that braking does not cost nor deliver energy to or from the catenary.

Three different driving strategies are compared in the model: time-optimal driving strategy (technical minimum running time), energy-efficient driving strategy (minimizing total traction energy), and the *UZI method*.

We consider time and speed as function of distance because then distance-related constraints, such as gradients, curves, tunnels and speed restrictions, are easily added to the model. The objective of energy-efficient train control is to minimize total traction energy consumption. This can be described as follows:

$$\min E_{mech} = \int_0^X F_{Tr}(x)dx,$$ (2.1)

where $E_{mech}$[Ws=J] is the mechanical energy needed to move the train, $F_{Tr}$ [N] is the traction force, $x$ [m] is the location of the train, and $X$ [m] is the total distance. The train motion can be described by two differential equations as function of distance $x$:

$$\frac{dt}{dx} = \frac{1}{v(x)},$$ (2.2)

$$\frac{dv}{dx} = \frac{F_{Tr}(v) - R(x,v)}{v \cdot \rho \cdot m},$$ (2.3)

where $R(x,v)$ [N] is the total resistance of the train consisting of train resistance and track resistance, $v(x)$ [m/s] is the speed of the train, $m$ [kg] is the total mass of the train, and $\rho$ the rotating mass factor. The total resistance is based on the Davis equation (Davis, 1926):

$$R(x,v) = R_0(x) + R_1 \cdot v + R_2 \cdot v^2,$$ (2.4)

where $R_0$ is a coefficient related to line resistance (like gradients and curves), $R_1$ to rolling resistance, and $R_2$ due to air resistance.

It is convenient to normalize the equations and make it mass specific. Therefore, we introduce the specific traction force $u(v)$ [N/kg]

$$u(v) = \frac{F_{Tr}(v)}{\rho \cdot m},$$ (2.5)

subject to

$$u(v)\epsilon[-u_{min}(v), u_{max}(v)],$$ (2.6)

with

$$u_{max}(v) = \frac{F_{Tr}^{max}(v)}{\rho \cdot m} > 0 \text{ and } u_{min}(v) = \frac{F_{br}^{max}(v)}{\rho \cdot m} > 0, \tag{2.7}$$

where $F_{br}$ [N] is the braking force of the train. And likewise we define the specific resistance $r(x, v)$ [N/kg] as

$$r(x, v) = \frac{R(x, v)}{\rho \cdot m} = r_0(x) + r_1 \cdot v + r_2 \cdot v^2. \tag{2.8}$$

Rewriting equation (2.3) with equations (2.5) and (2.8) leads to:

$$\frac{dv}{dx} = \frac{u(v) - r(x, v)}{v}. \tag{2.9}$$

## 3 Algorithm

This section will derive the energy-efficient train control based on the basic parts of the model in section 2 and propose an algorithm for the *EZR model*.

### 3.1 Optimal train control

The optimal train control problem is to find the control $u$ that minimizes the total energy given certain constraints. Since we do not consider regenerative braking, we are only interested in the positive traction. Therefore, we introduce the notation $u^+(x) = \max(u(x), 0)$. Then we can reformulate the objective of equation (2.1) with control variable $u$ to:

$$\min_u E_{mech}(u) = \int_0^X u^+(x)dx. \tag{3.1}$$

subject to

$$\frac{dt}{dx} = \frac{1}{v(x)}, \tag{3.2}$$

$$\frac{dv}{dx} = \frac{u(v) - r(x, v)}{v}, \tag{3.3}$$

$$t(0) = t_0, \quad t(X) = T, \tag{3.4}$$

$$v(0) = v_0, \quad v(X) = v_X, \tag{3.5}$$

$$v(x) \in [0, v_{max}(x)], \quad u(x) \in [-u_{min}(v(x)), u_{max}(v(x))]. \tag{3.6}$$

The optimal control $\hat{u}(x)$ can be found using Pontryagin's Maximum Principle, see e.g. Lewis (1986). For this, define the Hamiltonian as follows:

$$
\begin{aligned}
H(t,v,\lambda,\phi,u) & = -u^+ + \lambda \cdot \left(\frac{u}{v} - \frac{r(x,v)}{v}\right) + \frac{\phi}{v} \\
& = \begin{cases} u \cdot \left(\frac{\lambda}{v} - 1\right) - \frac{\lambda \cdot r(x,v)}{v} + \frac{\phi}{v} & \text{if } u \geq 0 \quad (3.7) \\ u \cdot \frac{\lambda}{v} - \lambda \cdot \frac{r(x,v)}{v} + \frac{\phi}{v} & \text{if } u < 0. \end{cases}
\end{aligned}
$$

where $\lambda(x)$ and $\phi(x)$ are co-state variables satisfying the differential equations:

$$
\begin{aligned}
\frac{d\lambda}{dx} & = -\frac{\partial H}{\partial v}(t,v,\lambda,\phi,\hat{u}) \\
& = \frac{\lambda(x) \cdot u(x) + \lambda(x) \cdot v \cdot \frac{\partial r}{\partial v}(x,v) - \lambda(x) \cdot r(x,v) + \phi(x)}{v^2}, \quad (3.8)
\end{aligned}
$$

$$
\frac{d\phi}{dx} = -\frac{\partial H}{\partial t}(t,v,\lambda,\phi,\hat{u}) = 0. \qquad (3.9)
$$

The last differential equation (3.9) implies that $\phi(x) = \phi_0$ is constant. According to Pontryagin's Maximum Principle the optimal control variable $\hat{u}(x)$ maximizes this Hamiltonian. The Hamiltonian is a linear function in $u$ for both positive and negative values of $u$. Hence, the coefficient of $u$ in the Hamiltonian determines the switching point where the sign determines which value $\hat{u}$ should have to maximize the Hamiltonian. For $u(x) \geq 0$ the switching function is $\sigma(x) = \lambda(x)/v(x) - 1$, and for $u(x) < 0$ the switching function is $\sigma(x) = \lambda(x)/v(\text{x})$. If the switching function is zero ($\lambda(x) = v(x)$ or $\lambda(x) = 0$) then the optimal control has a singular solution. The situation $\lambda(x) = 0$ cannot occur over an interval bigger than one point, so only the singular solution $\lambda(x) = v(x)$ remains. This leads to the optimal control $\hat{u}(x)$ that maximizes the Hamiltonian:

$$
\hat{u}(x) = \begin{cases} u_{max}(x) & \text{for } \lambda(x) > v(x) & \text{(Maximum acceleration)} \\ [0, u_{max}(x)] & \text{for } \lambda(x) = v(x) & \text{(Cruising)} \\ 0 & \text{for } \lambda(x) < v(x) & \text{(Coasting)} \\ -u_{min}(x) & \text{for } \lambda(x) \leq 0 & \text{(Maximum braking).} \end{cases} \qquad (3.10)
$$

Thus the optimal driving strategy consists of the following driving regimes, based on equation (3.10):
- Maximum acceleration
- Cruising
- Coasting, and
- Maximum service braking.

Figure 1 illustrates these driving regimes including the switching points, s1: switching point of acceleration to cruising, s2: switching point of cruising to coasting and s3: switching point from coasting to braking.
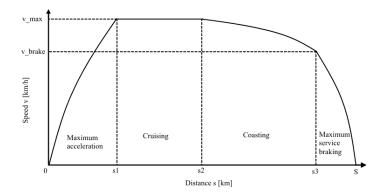
Figure 1: Optimal driving regimes for energy-efficient driving

The singular solution gives the optimal cruising speed which is somewhere between zero and the speed limit and which requires a traction force as a function of distance $x$ to maintain this cruising speed. In particular, the traction force must balance the resistance function $r(x, v)$. According to equation (3.10) the following holds in the cruising regime: $\lambda(x) = v(x) = v_{cruis}$, where $v_{cruis}$ is the optimal cruising speed, and moreover $\lambda'(x) = v'(x)$ so that the singular solution is maintained over some distance. Using (3.8) and (3.3) then gives

$$\frac{v_{cruis} \cdot u(x) + v_{cruis}^2 \cdot \frac{\partial r}{\partial v}(x, v_{cruis}) - v_{cruis} \cdot r(x, v_{cruis}) + \phi_0}{v_{cruis}^2} = \frac{u(x) - r(x, v_{cruis})}{v_{cruis}}.$$

(3.11)

Using $\frac{\partial r}{\partial v}(x, v) = r_1 + 2r_2 \cdot v$ and simplifying then gives

$$\phi_0 = -v_{cruis}^2(r_1 + 2r_2 \cdot v_{cruis}).$$ 

(3.12)

Equation (3.12) contains two unknown parameters ($v_{cruis}$ and $\phi_0$), so we need another equation to find the optimal cruising speed. According to Pontryagin's Maximum Principle the Hamiltonian is constant for the optimal trajectory if $u(x) > 0$. This leads to the following equation for the Hamiltonian (traction driving regime):

$$H(t, v, \lambda, \phi, u) = -u(x) + \frac{\lambda(x) \cdot u(x)}{v} - \frac{\lambda(x) \cdot r(x, v)}{v} + \frac{\phi_0}{v} = c_0.$$

(3.13)

The optimal cruising speed $v_{cruis}$ can then be determined as follows. First, equation (3.13) can be rewritten as

$$\phi_0 = c_0 \cdot v + v \cdot u(x) - \lambda(x) \cdot u(x) + \lambda(x) \cdot r(x, v)$$

(3.14)

and during cruising with $\lambda(x) = v(x) = v_{cruis}$ this translates to

$$\phi_0 = c_0 \cdot v_{cruis} + v_{cruis} \cdot r(x, v_{cruis}). \qquad (3.15)$$

Finally, combining equation (3.12) and (3.15) results in

$$\begin{aligned} c_0 &= -v_{cruis} \cdot (r_1 + 2r_2 \cdot v_{cruis}) - r_0(x) - r_1 \cdot v_{cruis} - r_2 \cdot v_{cruis}^2 \\[2mm] &= -r_0(x) - 2r_1 \cdot v_{cruis} - 3r_2 \cdot v_{cruis}^2 \end{aligned} \qquad (3.16)$$

Furthermore, equation (3.12) shows that $\phi_0 < 0$ and equation (3.16) shows that $c_0 < 0$. These results can be used to solve equation (3.12) to find the optimal cruising speed $v_{cruis}$.

### 3.2  Implementation

The main challenge is to determine the exact switching points, i.e., where the change from one driving regime to another one occurs. The reason for this is that there are only two equations (equations (3.12) and (3.15)) with at least three unknown variables ($v_{cruis}$, $\mu$ and $c_0$). Therefore the problem is solved constructively using the known optimal driving regimes (see equation (3.10) and Figure 1) by efficient search algorithms. When the optimal strategy is found, the value of co-state variable $\lambda(x)$ can be solved and the solution of $\lambda(x)$ can be checked by its graph and the results for $\lambda(x)$ from equation (3.10).

Therefore an algorithm has been developed that iteratively calculates the optimal cruising speed and coasting point with the objective to minimize total traction energy consumption (equation (3.1)). From the optimal control theory described above it is known that a train applies maximum acceleration and service braking for minimizing total traction energy. Because of this, the acceleration and braking curves are known. Secondly the train resistance and the track resistance are known for the trajectory where the train is running. With this the coasting curve can be determined. Therefore the point where the train changes driving regime from acceleration to cruising is fixed, if the cruising speed is known. Besides, the point where the train changes from coasting to braking is fixed, because this is the intersection of the braking curve with the coasting curve. The only variable point is the switching point from cruising (with long distance between two consecutive stops) or acceleration (short stops) to coasting.

The algorithm starts with determining the coasting regime. Based on an initial estimate of the distance where the coasting regime starts, the algorithm calculates the speed profile and total travel time to go from the start point toward the end point using differential equations (2.9) and (2.5). The total available running time between two stations is known from the timetable (technical minimum running time plus running time supplements). Therefore the needed running time can be compared with the available running time. The error between them enables a search where the difference becomes zero (required time is equal to available time). This is done by the bisection method, which is described in for example Hillier and Lieberman (2015). This way it is possible to adjust the next guess for the starting point of coasting. This bisection method continues until the optimal coasting point has been calculated within a small enough error.

Second, the algorithm determines the cruising phase. If there are no speed limits then the objective is to find the optimal cruising speed. But also in the situation with running time supplements and constraints, the energy-efficient train driving strategy can consist of a cruising speed lower than the maximum speed limit. Only if the optimal cruising speed

exceeds the speed limit, the maximum speed of the train is limited to the speed limit. The optimal cruising speed depends on the geometrical track characteristics and the given time supplement. The algorithm determines the optimal cruising speed by the Fibonacci search algorithm. The Fibonacci search algorithm is described by for example Mathews and Fink (2004). The objective function is to minimize the total traction energy consumption. The method uses the Fibonacci numbers to efficiently calculate the minimum of the objective function. The upper bound of the search algorithm is the maximum allowed speed. The lower bound of the algorithm is calculated by searching the slowest speed of the train with the driving regimes acceleration, braking and if possible cruising to be on time according to the timetable. With these bounds the Fibonacci algorithm calculates the running time based on a golden ratio search.

The energy-efficient speed profile is now calculated using two loops: the outer loop of the Fibonacci algorithm finds the optimal cruising speed while in an inner loop the bisection method finds the optimal coasting point for the given cruising speed. The algorithm is illustrated in Figure 2.

The model has been implemented in MATLAB. The model can be described in a basic input-model-output format. A visualization of this implementation is shown in Figure 3. The MATLAB model consists of three different parts that calculate the minimum running time (time-optimal) driving strategy, the *UZI method* and the energy-efficient driving strategy. One of the outputs of the model is the co-state variable $\lambda(x)$. This output is only generated for the test runs, to check if the developed algorithm generates results that are consistent with the theory. The co-state variable is plotted over distance to check the switching points of the optimal driving strategy.
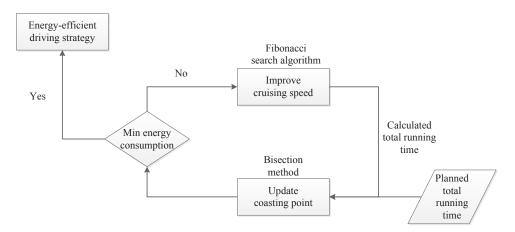


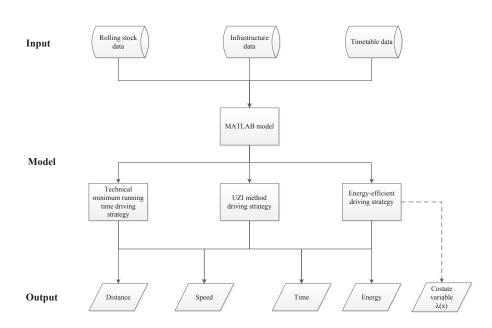Figure 2: Overview algorithm to determine optimal train control

Figure 3: Input-model-output overview of *EZR model*

## 4   Results

This section describes the *EZR model* results. First, the data input and the model calibration is described. After that, the model is applied on the local train line between Utrecht Centraal and Rhenen with the current running time supplement distribution from the timetable. The model results are compared with the *UZI method* to verify the model. Moreover, the model results are compared with punctuality results from practice in order to validate the model. Then the running time supplements are uniformly redistributed in order to increase punctuality and to determine the effect on the total energy consumption. Finally, real-time test run results of the *EZR model* are shown.

### 4.1  Input and calibration

The model has been built in MATLAB step by step and successively calibrated, validated and verified. Then the model has been applied to a case study. For this case study the local/sprinter train line 7400 was selected between the stations Utrecht Centraal and Rhenen. Figure 4 shows this network. Only the direction of Utrecht Centraal to Rhenen was considered. The trains on this line serve all intermediate stations. The corridor Utrecht Centraal – De Haar junction is operated by mixed traffic (i.e. high speed, intercity, sprinter and freight trains) while the corridor De Haar junction – Rhenen is operated by sprinter trains only. The rolling stock on this line is mainly *Sprinter Light Train* (SLT), see Figure 5. The timetable used for the case study was the Dutch timetable of 2012. The model is used to compare three different driving strategies with each other, i.e. time-optimal (technical minimum running time), energy-efficient (minimization of total traction energy without regenerative braking) and the *UZI method*. For the case study the following data were used:

- Rolling stock: six-car SLTs of NS. The technical data is obtained from TreinPlein (NSR/Lloyd's, 2012).
- Infrastructure: the infrastructure between Utrecht Centraal and Rhenen in 2012 is obtained from the InfraAtlas database of ProRail. This includes the signalling system and the Dutch train protection system ATB (in Dutch 'Automatische TreinBeïnvloeding'). Since from the track resistance components only the gradient profile is provided in InfraAtlas, only the gradients are considered in the model (no curves). The track resistances can be found in Figure 6.
- Timetable: the Dutch timetable of 2012 for train line 7400 obtained from the Dutch timetable design tool Donna. In this case only a single train is considered, but other trains are implicitly taken into account by certain passing times at important timetable points (like De Haar junction).
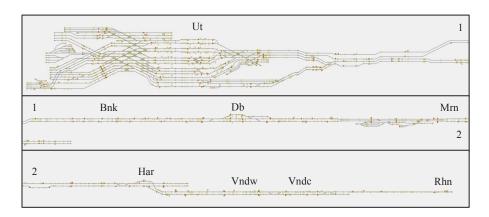


Figure 4: Overview case study local train between Utrecht Centraal (Ut) and Rhenen (Rhn)



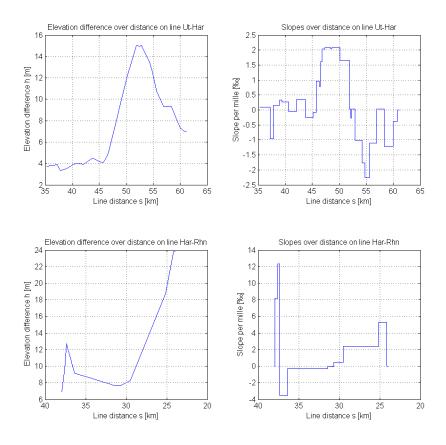Figure 5: *Sprinter Light Train* (SLT) (left) and VIRM (right) of NS

Figure 6: Gradient profile on the lines Ut-Har (above) and Har-Rhn (below)

This railway line contains the following timetable points:
- Station Utrecht Centraal (Ut);
- Station Bunnik (Bnk);
- Station Driebergen-Zeist (Db);
- Station Maarn (Mrn);
- Junction De Haar (Har);
- Station Veenendaal West (Vndw);
- Station Veenendaal Centrum (Vndc);
- Station Rhenen (Rhn).

First of all the model is calibrated on the basis of rolling stock type VIRM 1 IV (see Figure 5). The model results are compared with the data from Lloyd's Register Rail Europe B.V. They performed some energy calculations for different driving regimes for different types of trains commissioned by NS in 2009. The results of the model seem to fit the data from Lloyd's Register Rail. The greatest deviations are caused by the fact that data of the rolling stock from the TreinPlein data are based on new coefficients for the traction force and the train resistance, while the data of Lloyd's Register Rail is based on old coefficients. Extended research can be found in Scheepmaker (2013).
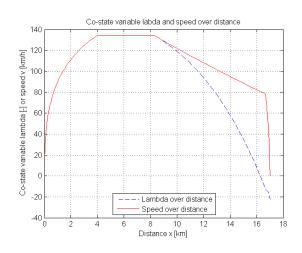
Figure 7: Co-state variable $\lambda$ and speed over distance

Second, the algorithm is checked with the theory by calculating the co-state variable $\lambda(x)$ after the solution was found. This is done on a fictive simple test case with track length of 17 km without elevation differences, curves and tunnels on the track, rolling stock type VIRM 1 IV and total available running time of 600 s. Since it was not possible to solve two equations ((3.12) and (3.15)) with at least three unknown variables ($v_{cruis}$, $\phi_0$ and $c_0$), the optimal cruising speed from the model is used to check whether the results match with the optimal control theory. After the model has run, the parameter $v_{cruis}$ is known, so it is possible to calculate the other variables and to solve the differential equation of the co-state variable $\lambda(x)$ in equation (3.8). This calculation is started at the end of the coasting phase until the end of the braking phase. Equation (3.10) states that the start of the coasting phase $\lambda(x) = v_{cruis}$ and at the switching point form the coasting to braking phase $\lambda(x) = 0$ holds. The plot of the co-state variable $\lambda$ can be found in Figure 7. The developed model has an error of approximately 2.4% (or 389 m) compared to the optimal switching point, which was considered sufficiently close to conclude that the *EZR model* approaches the optimal control satisfactory.

The computation time of the model may still be improved. Without a varying gradient profile the model calculates the optimal solution within 15 seconds. However, the calculation of a track with gradient profile takes about 90 seconds and even 190 seconds when including the signalling and train protection system. Moreover, the model only calculates the optimal driving strategy between two points with constant speed limit. So a complete line requires multiple runs. However, since the *EZR model* is not used as a DAS, the speed of the model is not essential at this time.

## 4.2  Current running time supplement distribution

Alongside the calibration of the model, the model is also verified. The verification includes analysing to which degree the results of the model match with reality and how the energy-efficient driving strategy is related to the *UZI method*. This shows that the model delivers results that meet the expectations and the algorithm reaches the theoretical optimum very close. The model is applied on the section Utrecht Centraal – Rhenen. Given the timetable from Donna there are only a few parts along the complete line where it is possible to drive energy-efficient due to the distribution of the running time

supplements, see Figure 8 and Figure 9. In these figures the black dotted line represents the maximum track speed. The energy-efficient driving strategy supplies an energy saving of 15.7% compared to the time-optimal driving strategy on the complete section Utrecht Centraal – Rhenen, which can be found in Table 2 and Table 3. The *UZI method* makes it possible to save 12.1% compared to the time-optimal strategy on the complete section. In theory extra energy savings of 4.1% are possible on the complete section if the energy-efficient driving strategy is used instead of the *UZI method*.

A validation has been carried out to test whether the model results match the results from practice (Scheepmaker, 2013). This analysis revealed that the model delivers results which have a good match with results from practice. The model results and the bandwidth graph of punctuality prove that the time-optimal driving strategy from Donna is not feasible, see Figure 10. In this figure the punctuality of train line 7400 uneven between Breukelen (Bkl), Utrecht Centraal (Ut) and Rhenen (Rhn) is visualized. The red line is the 50 percentile, the thick blue lines are the 10 (below) and 90 (above) percentiles. The other thin blue lines are the other dozens of percentiles. On the horizontal axis the figure shows the arrival (A), the passing (D) and departure (V) times of the trains at locations where the punctuality is measured. The figure indicates that trains arrive too late at station Driebergen-Zeist (Db). The same follows from the analysis of the running times and the timetable from Donna. Donna generates running times which in general are lower than the ones the model predicts. A reason could be the fact that Donna does not take into account track resistances for passenger trains, while the model does take them into account. Donna is indeed still not validated.
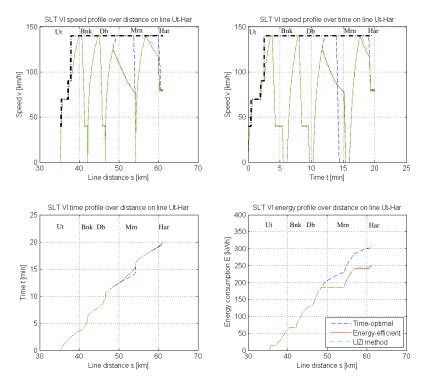


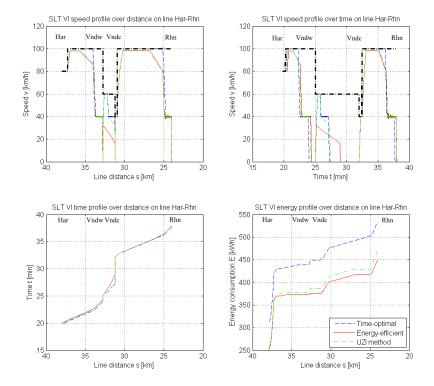Figure 8: Speed profiles, time profile and energy profile on line Ut – Har

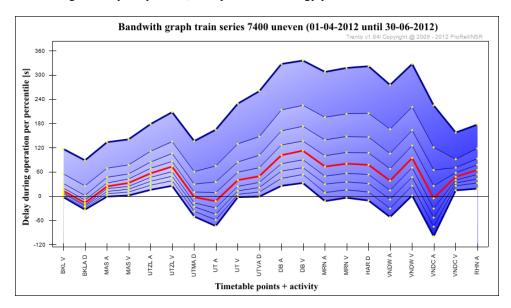Figure 9: Speed profiles, time profile and energy profile on line Har – Rhn



Figure 10: Bandwidth graph of punctuality of train line 7400 (uneven) on Bkl-Ut-Rhn

### 4.3 Uniform redistribution of running time supplements

The last section showed that the current running time distribution causes punctuality problems on intermediate stations and energy-efficient train operation is only limited possible. Therefore, we redistributed the running time supplements uniformly over the complete line in order to apply energy-efficient train operation on more sections. Again the possibilities of energy-efficient operation have been investigated, without hindering surrounding trains. The results of the analysis show that extra savings (above the savings mentioned before) are possible of 7.2% for the energy-efficient driving strategy and 4.3% for the *UZI method* on the section Utrecht Centraal – Rhenen. The results are also visualized in Figure 11 and Figure 12 and summarized in Table 2 and Table 3 calculated with equation (3.1). This analysis shows that tightening of the timetable is unfavourable for energy-efficient train operation.

Table 3 also reveals that the *UZI method* applied to the timetable with uniform distributed running time supplements performs much better than on the existing timetable. Indeed, the *UZI method* assumes that 5% running time supplement is available between two successive stops. Moreover, even the energy-efficient train control applied to the existing timetable performs less than the *UZI method* on the uniform timetable. Hence, improving the distribution of running time supplements in the current timetable philosophy leads to better results than investing in advanced train control systems and with much less investments. Or the other way around: it is not useful to invest in driver advisory systems with optimal energy-efficient driving functionalities if the timetable design process follows the tightening approach. But with uniformly distributed running time supplements the optimal energy-efficient driving advisory systems lead to considerable energy savings.

Table 2: Energy consumption per driving strategy and timetable based on equation (3.1)

| Timetable | Time-optimal [kWh] | UZI method [kWh] | Energy-efficient [kWh] |
|---|---|---|---|
| Existing | 533.0 | 468.4 | 449.1 |
| Uniform | 533.0 | 448.3 | 416.9 |

Table 3: Energy savings per train run relative to time-optimal

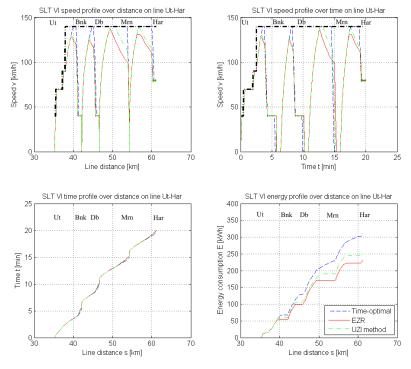| Timetable | UZI method | Energy-efficient |
|---|---|---|
| Existing | 64.6 kWh (12.1%) | 83.9 kWh (15.7%) |
| Uniform | 84.7 kWh (15.9%) | 116.1 kWh (21.8%) |

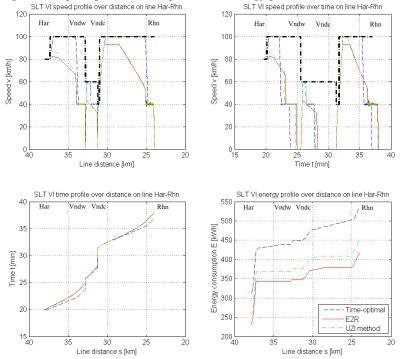Figure 11: Speed profiles, time profile and energy profile on line Ut – Har

Figure 12: Speed profiles, time profile and energy profile on line Har – Rhn
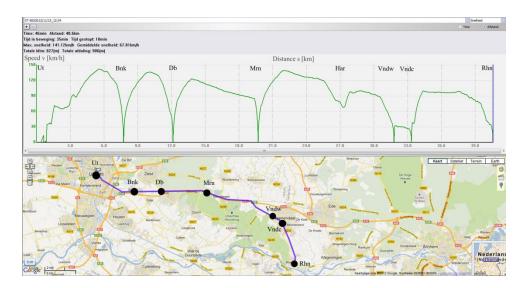
Figure 13: Result real-time test run with speed profile (measured with GPS)

## 4.4 Real-time test run

The model was also applied to reality for train 7443 on 15-11-2012. In order to do this the model results have been translated into a service card used by train drivers (timetable for the train driver). The test result shows that the model is working in practice since all intermediate timetable points are reached on time (realizable timetable and punctual execution). A GPS measurement of the test run can be found in Figure 13. However, the train resistance and braking position 1 according to the Dutch ATP (Automatic Train Protection) system may have been chosen too conservative. In reality a train coasts longer than the model has computed, and the braking force in position 1 is in practice less vigorous than the model computes. In addition to this there were almost ideal conditions during the practical measurement, while the parameters of the rolling stock also take into account less favourable conditions. Unfortunately, a test run with only one train could be done and so the statements may not be generalized. Detailed information about the test run can be found in Scheepmaker (2013).

## 5   Conclusions

An energy-efficient train control model has been developed in MATLAB and has been applied to a real case. This so-called *EZR model* determines the energy-efficient driving strategy by calculating the optimal cruising speed and coasting point based on the knowledge of the optimal energy-efficient driving regimes obtained from Pontryagin's Maximum Principle. The model gives results in terms of energy use which are quite close to the theoretical optimum. Thereby the model takes into account the desirable robustness and the desirable punctuality during the execution of the timetable. The results of the research based on the case study show that there are significant energy savings possible by using the *UZI method* instead of the time-optimal driving strategy (technical minimum running time) (12.1% energy savings). These energy savings can be larger when using the energy-efficient driving strategy (15.7%). So, extra energy savings of 4.1% can be achieved by using the energy-efficient driving strategy instead of the *UZI method* on the

section Utrecht Centraal – Rhenen for local train line 7400. Moreover the results show that using a uniform distribution of the running time supplements leads to extra energy savings and an improvement on punctuality compared to the method of tightening the timetable. The results show that 7.2% extra savings can be achieved with the energy-efficient driving strategy for the uniform distribution instead of the current distribution. For the *UZI method* a saving of 4.3% is possible.

Therefore this research shows that more benefits can be reached when improving the timetable (i.e. running time supplement distribution) than by improving the energy-efficient driving strategy of trains (like a DAS). Hence, further research on the topic of optimal running time supplement distribution in railway timetables is recommended. Of course, a joint optimization of the timetable and energy-efficient driving leads to the most energy savings. Another topic for future research is to incorporate regenerative braking into the optimal control strategy and to see how this influences the driving strategy. Finally, further research on the topic of easiness of driving of the optimal driving strategy is recommended.

## References

Albrecht, A. R., Howlett, P. G., Pudney, P. J., & Vu, X. (2013). Energy-efficient train control: From local convexity to global optimization and uniqueness. *Automatica, 49*(10), 3072-3078.

Albrecht, T. (2008). Energy-Efficient Train Operation. In I. A. Hansen & J. Pachl (Eds.), *Railway Timetable & Traffic* (pp. 83-105). Hamburg, Germany: Eurailpress.

Albrecht, T., & Oettich, S. (2002). A new integrated approach to dynamic schedule synchronization and energy-saving train control. In J. Allen, R. J. Hill, C. A. Brebbia, G. Sciutto & S. Sone (Eds.), *Computers in Railways VIII* (pp. 847-856). Southampton, United Kingdom: WIT Press.

Brünger, O., & Dahlhaus, E. (2008). Running Time Estimation. In I. A. Hansen & J. Pachl (Eds.), *Railway Timetable & Traffic*. Hamburg, Germany: Eurailpress.

Chevrier, R., Pellegrini, P., & Rodriguez, J. (2013). Energy saving in railway timetabling: A bi-objective evolutionary approach for computing alternative running times. *Transportation Research Part C: Emerging Technologies, 37*(0), 20-41.

Davis, W. J. (1926). The tractive resistance of electric locomotives and cars. *General Electric Review, 29*(October).

Domínguez, M., Fernández-Cardador, A., Cucala, A. P., & Pecharromán, R. R. (2012). Energy Savings in Metropolitan Railway Substations Through Regenerative Energy Recovery and Optimal Design of ATO Speed Profiles. *Automation Science and Engineering, IEEE Transactions on, 9*(3), 496-504.

Franke, F. (2012). Energy-efficient operation (in Dutch: Energiezuinig Rijden). Utrecht, the Netherlands: NS Reizigers.

Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to Operations Research*. New York, NY, United States of America: McGraw Hill Education.

Howlett, P. G. (1990). An optimal strategy for the control of a train. *The ANZIAM Journal, 31*(04), 454-471.

Howlett, P. G. (2000). The Optimal Control of a Train. *Annals of Operations Research, 98*(1), 65-87.

Howlett, P. G., & Pudney, P. J. (1995). *Energy-Efficient Train Control*. London, United Kingdom: Springer-Verlag.

Howlett, P. G., Pudney, P. J., & Vu, X. (2009). Local energy minimization in optimal train control. *Automatica, 45*(11), 2692-2698.

Kent, S. (2009). Driver advisory information for energy management and regulation Stage 1 Report - Appendices. London, United Kingdom: DeltaRail.

Khmelnitsky, E. (2000). On an optimal control problem of train operation. *Automatic Control, IEEE Transactions on, 45*(7), 1257-1266.

Lewis, F. L. (1986). *Optimal Control*. New York, NY, United States of America: Wiley.

Liu, R. R., & Golovitcher, I. M. (2003). Energy-efficient operation of rail vehicles. *Transportation Research Part A: Policy and Practice, 37*(10), 917-932.

Mathews, J. H., & Fink, K. D. (2004). *Numerical Methods Using MATLAB*. Upper Saddle River, N.J., United States of America: Pearson Prentice Hall.

NSR/Lloyd's (2012). TreinPlein.  Retrieved 1 June, 2012, from treinplein.net.

Panou, K., Tzieropoulos, P., & Emery, D. (2013). Railway driver advice systems: Evaluation of methods, tools and systems. *Journal of Rail Transport Planning & Management, 3*(4), 150-162.

Scheepmaker, G. M. (2013). *Running time supplements in railway timetables: Energy-efficient operation versus robustness (in Dutch: Rijtijdspeling in treindienstregelingen: energiezuinig rijden versus robuustheid).* (MSc Thesis), NS Reizigers/Delft University of Technology (TU Delft), Utrecht, the Netherlands.

Sicre, C., Cucala, A. P., & Fernández-Cardador, A. (2014). Real time regulation of efficient driving of high speed trains based on a genetic algorithm and a fuzzy model of manual driving. *Engineering Applications of Artificial Intelligence, 29*(0), 79-92.